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Turbulence Amplification in Shock-Wave Boundary-Layer Interaction

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Attention is directed to the acoustics research of the 1950s and 1960s for guidance in understanding and quantizing the turbulence amplification that can occur in regions of shock-wave boundary-layer interaction. Three primary turbulence amplifier-generator mechanisms are identified and shown, by linear analysis, to be responsible for turbulence amplification across a shock wave in excess of 100% of the incident turbulence intensity.

Nomenclature

A'	= amplitude of incident plane harmonic wave
c_1,c_2	= speed of sound ahead of and behind shock, respectively
c_p	= specific heat at constant pressure
F_1	$=(M_{2n}/M_{1n})=[\{2+M_{1n}^2(\gamma-1)\}/\{M_{1n}^2 \times(2\gamma M_{1n}^2-(\gamma-1)\}]^{1/2}$
F_2	$=(\bar\rho_2/\bar\rho_1)=(\gamma+1)M_{1n}^2/\{2+M_{1n}^2(\gamma-1)\}$
F_3	$=(\bar p_2/\bar p_1)=\{2\gamma M_{1n}^2-(\gamma-1)\}/(\gamma+1)$
F_4	$=(c_2/c_1)=(F_3/F_2)^{1/2}$
F_5	$=c_2^2M_{2n}^2P=(\gamma-1)(M_{1n}^2-1)^2/\{M_{1n}^2(\gamma+1)\}$
F_6	$=\{1+2\alpha_2F_1M_{1n}^3+M_{1n}^2\}/M_{1n}^2$
F_7	$=(1-u_yk_y/\omega)\{\alpha_3F_6+2\beta_2\beta_3F_1M_{1n}\}-(c_1k_y/\omega)\{\beta_3(M_{1n}^2-1)/M_{1n}\}$
$G_P,G_{\tilde p},G_E,G_V$	= turbulence plane wave transfer/generation fuctions, as defined in text
H_1	$=F_6M_{1n}(c_1k_y/\omega)$, definition
H_2	$=(1-u_yk_y/\omega)\beta_2+(c_1k_y/\omega)\times\{(1+\alpha_2F_1M_{1n}^3)/(F_1M_{1n}^2)\}$, definition
H_3	$=F_6(1-u_yk_y/\omega)$, definition
k	= disturbance plane wave vector
M_1,M_2	= unperturbed stream Mach number ahead of and behind shock, respectively
M_{1n},M_{2n}	= unperturbed stream Mach number normal to shock line, ahead of and behind shock, respectively
p	= local pressure
$\tilde p'$	= amplitude of plane acoustic wave incident from behind shock
P,Q	= parameters as defined in Eqs. (13b) of text
s	= local entropy
u_s	= amplitude of shock oscillation parallel to x axis
u	= local velocity
u',v'	= components of plane vorticity wave velocity vector behind shock, parallel and normal to unperturbed stream
U'	= rms value of fluctuating velocity vector $(\equiv\sqrt{u'^2+v'^2})$
w	= local enthalpy
x,y	= coordinate directions normal and parallel to unperturbed shock line, respectively
x',y'	= coordinate directions normal and parallel to unperturbed stream direction ahead of shock, respectively
α_i	$=\cos\theta_i$, $i=1,2,3$
β_i	$=\sin\theta_i$, $i=1,2,3$
δ	$=+1$ for fast acoustic wave, -1 for slow acoustic wave
δ^*	= angle of deflection of unperturbed stream by shock
ρ	= local density
γ	= specific heats ratio
θ_1	= angle of incidence of plane wave relative to unperturbed shock normal
θ_2	= angle of divergence of plane acoustic wave refracted or generated behind a shock
θ_3	= angle of divergence of plane entropy or vorticity wave refracted or generated behind a shock
θ_s	= shock-wave angle
Θ	$=\theta_s$ ahead of shock, $(\theta_s-\delta^*)$ behind shock
ω	= plane disturbance wave frequency
$(\ )'$	= magnitude of a fluctuation quantity
$\bar{(\ )}$	= magnitude of an unperturbed quantity
$(\ )_x,(\ )_n$	= magnitude of vector component normal to unperturbed shock
$(\ )_y,(\ )_T$	= magnitude of vector component parallel to unperturbed shock
$(\ )_1,(\ )_2$	= values ahead of and behind shock, respectively (except as otherwise indicated)

Introduction

It is often necessary to compute the region of interaction between a shock wave and a turbulent boundary layer, particularly in supersonic inlet and transonic airfoil design problems. Because of the usually intense flowfield gradients in such problems, the full Navier-Stokes (N-S) equations must be solved. For cases where flow separation occurs, treatment of the shear stress term appears to be especially critical to the quality of the computation.

Attempts to extend low-speed turbulence closure methods to high-speed boundary-layer calculations have been quite successful in many instances, even up to Mach numbers the order of 20.[1] But such efforts can fail miserably when extended to the computation of the region of interaction between the shock wave and turbulent boundary layers, and especially so for separated flow cases where the mean flow is no longer largely pressure driven.[2,3] We typically attribute this failure to the extreme rapid distortion of the flowfield which characterizes the region of shock-wave boundary-layer interaction. Such rapid distortion appears to negate equilibrium assumptions upon which most low-speed turbulence models are based.

Some nonequilibrium turbulence closure methods based upon rapid distortion theory[4] have already been developed,

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for instance in Ref. 5. Low-speed results,[6,7] for abrupt enough disturbance of a turbulent flow, indicate the Reynolds stress remains nearly frozen at its initial value while it is being convected along streamlines across the region of the abrupt disturbance. However, experiments indicate large amplifications of Reynolds stress and turbulence intensity across the shocked region of a high-speed turbulent boundary layer.[8-11] Therefore, some new physics not captured by current rapid distortion theories, and probably associated with compressibility aspects, must be sought for problems of shock-wave/boundary-layer interaction.

Theoretical predictions of such turbulence amplifications had already been presented as far back as the 1950s, especially by Ribner,[12,13] but did not seem to have been noticed by fluid dynamicists, perhaps because Ribner's and similar work[14-16] were presented in the context of acoustics research. Ribner's turbulence amplification calculations and a related one on shock-vortex interaction[17] have since been confirmed by experiments.[10,24] The inescapable indications of these experimental and theoretical considerations and shock-wave/boundary-layer interactions are: that several mechanisms exist to amplify turbulence intensity and Reynolds stress across a shock and that the operation of these mechanisms may be reasonably predicted by linearized perturbation of the Rankine-Hugoniot jump conditions.

Our objective in this paper is primarily to reidentify some of the more important mechanisms that appear to influence the evolution of turbulence across shocked regions of shear layers, and the respective degrees of the influences of such mechanisms. We address particularly the hope that by focusing on a few of the more important turbulence amplifier-generator mechanisms, one may employ linear jump relations across the shocked region of a turbulent boundary layer to develop amplification factors, or transfer functions, for the shock-wave influence upon turbulence. The resultant turbulence intensity levels downstream of the shock interaction region could then be employed as initial conditions for a more conventional relaxation solution (albeit probably with an altered spectra necessitating altered constants).

Theoretical Foundation

The most general motion of a deformable body is characterized by, among other things, three fluctuation fields: 1) acoustic (fluctuating pressure and irrotational velocity mode); 2) turbulence (fluctuating vorticity mode); and 3) entropy (fluctuating temperature or, at constant pressure, fluctuating density mode). Theoretically, these three fluctuation modes are nonlinearly coupled, although they may appear to be independent when the fluctuation intensities are very small or when the mean flowfield distortion is relatively weak. For passage through a shock wave the coupling relations among the three fluctuation modes can readily be extracted from a linearized perturbation of the Rankine-Hugoniot jump conditions, if one presumes that the fluctuation intensities are weak relative to the mean flowfield distortion intensity. Such relations clearly indicate that in a compressible shear flow, for instance, whenever any one of the three fluctuation modes (noise, turbulence, or entropy) transfers across a shocked region, it not only generates the other two fluctuation modes but may itself also be significantly amplified behind the shocked region. Several theoretical studies of this phenomenon have been made: Ribner,[12,13] Chang,[15] Morkovin,[14] McKenzie and Westphal,[19] to name but a few. For the most, they all agree in the predictions of fluctuation amplification and generation across the region of interaction between a shock wave and a stream of a perfect gas. But we have found the approach presented by McKenzie and Westphal to be mathematically very simple and computationally easy to implement.

We are specifically interested in the turbulence fluctuation mode. Therefore, we examine only the transfer of turbulence

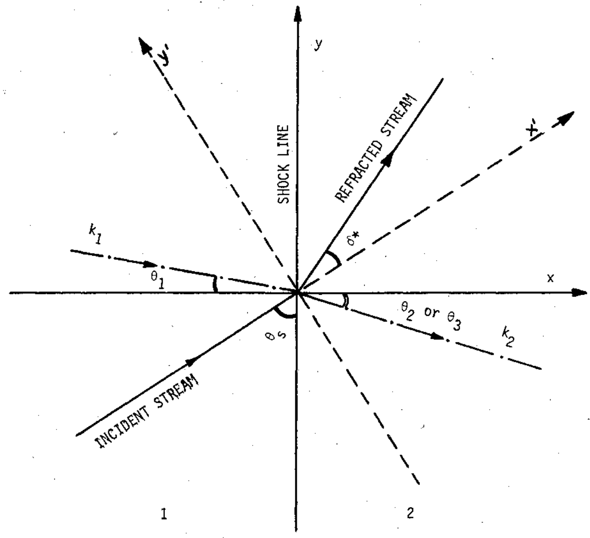

Fig. 1 Reference coordinate systems.

across a shocked region, and the generation of turbulence behind a shocked region resulting from the transfer of acoustic and entropy fluctuation modes across the shock. In the following sections we consider the relevant mathematical formulation and solutions of the problem, following the paper of McKenzie and Westphal.[19]

Basic Relations and Assumptions

We recognize that any small disturbance interacting which a shock wave may be generally decomposed into an angular spectrum of plane waves, each of which interacts with the shock wave independently of the others. Thus, we may first consider the problem of transferring across a shock wave, in an arbitrary medium, a single frequency plane wave disturbance of small-amplitude incident at some arbitrary angle upon a plane, stationary, oblique shock wave. An integration over all frequencies and all incident angles in the plane then provides the first-order transfer and/or generation functions for turbulence/shock-wave interaction.

We shall consider the incident perturbations to be plane harmonic waves of the form $A'\exp[i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]$ where A' is the small-perturbation amplitude.

The acoustic fluctuation mode will be propagated with the speed of sound c relative to the medium, while the entropy and turbulence modes will be convected with the medium. Thus, with reference to the nomenclature and the notations of Fig. 1 we have the following relations for the plane disturbance waves:

Acoustic Wave

$$p'\neq 0;\quad s'=0$$

$$U'=(p'/\bar{\rho}c)\{\alpha,\beta\}$$

$$\rho'=p'/c^2$$

$$\omega-\boldsymbol{u}\cdot\boldsymbol{k}=\pm ck \tag{1}$$

Entropy wave

$$s'\neq 0;\quad U'=p'=0$$

$$\rho'=rs'\equiv(\partial\bar{\rho}/\partial\bar{s})_{\bar{p}}\cdot s'$$

$$\omega-\boldsymbol{u}\cdot\boldsymbol{k}=0 \tag{2}$$

Turbulence (or vorticity) wave

$$U'=U'\{-\beta,\alpha\}\neq 0$$

$$p'=s'=\rho'=0$$

$$\omega-\boldsymbol{u}\cdot\boldsymbol{k}=0 \tag{3}$$

For waves of a single frequency the dispersion relations given above yield the magnitude of the wave vector $\boldsymbol{k}$ as a function of the wave direction and of the speed $\boldsymbol{u}$ of the medium. Thus, we have for

Acoustic waves

$$ck/\omega = 1/[\delta + M\sin(\Theta+\theta_1)] \quad (4)$$

where

$$\bar{u}_y k_y/\omega = M_1 \cos\Theta \beta_1 \delta/[\delta + M_1 \sin(\Theta+\theta_1)]$$

$$c_1 k_y/\omega = \beta_1/[\delta + M_1 \sin(\theta_s+\theta_1)]$$

Entropy and vorticity waves

$$ck/\omega = 1/[M\sin(\Theta+\theta_1)] \quad (5)$$

where

$$\bar{u}_y k_y/\omega = \beta_1 \cos\theta_s/\sin(\theta_s+\theta_1)$$

$$c_1 k_y/\omega = \beta_1/[M_1 \sin(\theta_s+\theta_1)]$$

The boundary conditions are the so-called Rankine-Hugoniot jump relations across a shock, namely

$$[\rho u_n]=0, \quad [p+\rho u_n^2]=0$$
$$[u_T]=0, \quad [w+u_n^2/2]=0 \quad (6)$$

where [ ] denotes the jump in a quantity across the discontinuity, for example, $[\rho u_n]=(\rho_1 u_{1n}-\rho_2 u_{2n})$.

Small-Perturbation Phase Jump Relation

When a plane disturbance wave crosses a shock the incident perturbation wave vector is refracted across the shock and diverges behind the shock at an angle different from the incident angle; furthermore, the other two disturbance wave modes generated behind the shock exhibit divergence. We presume that the conditions to be satisfied by the phases of the incident and diverging waves are that: the frequency ω and the component of the disturbance wave vector $\boldsymbol{k}$ tangential to the shock wave shall be continuous. We shall further assume that since we are dealing with small-amplitude perturbations, we may apply the continuity of ω and $\boldsymbol{k}$ relations at the plane of the unperturbed shock without serious error. Thus, in general we may write

$$\omega(\text{incident wave}) = \omega(\text{diverging wave})$$
$$k_y(\text{incident wave}) = k_y(\text{diverging wave}) \quad (7)$$

for any transmitted disturbance wave or between an incident and a generated disturbance wave across a shock. The tangential components of the wave vectors (k_y) are obtained for each wave type from the dispersion relations in Eqs. (4) and (5).

If we denote by θ_1 the angle of incidence of any perturbation wave, θ_2 the angle of divergence behind the shock of an acoustic wave, and θ_3 the angle of divergence behind the shock of an entropy or a vorticity wave, all angles being measured relative to the direction normal to the unperturbed shock, then implementation of the phase relations in Eq. (7) yields

1) For a fast acoustic wave incident from ahead of the shock

$$\cos\theta_2 = G_1 = [-R^2\beta_1^2 M_{2n}\delta + [(1+\alpha_1 M_{1n})^2\{(1+\alpha_1 M_{1n})^2 - R^2\beta_1^2(1-M_{2n}^2\delta)\}]^{1/2}]/\{R^2\beta_1^2 M_{2n}^2 + (1+\alpha_1 M_{1n})^2\} \quad (8a)$$

$$\tan\theta_3 = RM_{2n}\beta_1\delta/(1+\alpha_1 M_{1n}\delta) \equiv G_2 \quad (8b)$$

with $\delta = +1$ and $R = c_2/c_1$.

2) For a slow acoustic wave incident from ahead of the shock

$$\cos\theta_2 = G_1 \, (\text{with } \alpha_1 = -\alpha_1; \beta_1 = -\beta_1) \quad (9a)$$

$$\tan\theta_3 = G_2 \, (\text{with } \delta = -1) \quad (9b)$$

3) For an acoustic wave incident from behind the shock

$$\cos\theta_2 = G_1 \, (\text{with } R=1; M_{1n}=M_{2n}) \quad (10a)$$

$$\tan\theta_3 = G_2 \, (\text{with } R=1; M_{1n}=M_{2n}) \quad (10b)$$

4) For an entropy or a vorticity wave incident from ahead of the shock

$$\cos\theta_2 = [-R^2\beta_1^2\delta M_{2n} + [\alpha_1^2 M_{1n}^2\{\alpha_1^2 M_{1n}^2 - R^2\beta_1^2(1-M_{2n}^2)\}]^{1/2}]/(R^2\beta_1^2 M_{2n}^2 + \alpha_1^2 M_{1n}^2) \quad (11a)$$

$$\tan\theta_3 = RM_{2n}\tan\theta_1/M_{1n} \quad (11b)$$

In each case, there is a critical incident angle beyond which the acoustic wave divergence angle θ_2 becomes imaginary. Further consideration of the implications of this critical incidence may be found in Refs. 19 and 20.

Small-Perturbation Amplitude Jump Relations

A small-amplitude perturbation of the Rankine-Hugoniot jump conditions yields the following linearized jump relations for small-amplitude fluctuations across a shock wave

$[\bar{\rho}u_n' + \bar{u}_n\rho'] = 0$	continuity	(12)
$[p' + 2\bar{\rho}\bar{u}_n u_n' + \bar{u}_n^2\rho'] = 0$	normal momentum	
$[u_T'] = 0$	tangential momentum	
$[w' + \bar{u}_n u_n'] = 0$	energy	

If we express the internal energy in terms of pressure and density and if we assume an equation of state of the form, $\rho = \rho(p,s)$, then the concept of the shock adiabatic may be employed to replace the perturbed energy equation (12) with the following small-perturbation "pressure-entropy-density" relation

$$p_2'(c_2^{-2} - Q) + r_2 s_2' = Pp_1' + Wr_1 s_1' \quad (13a)$$

where

$$Q = (\partial\rho_2/\partial p_2)$$
$$P = (\partial\rho_2/\partial p_1) + (\partial\rho_2/\partial\rho_1)/c_1^2$$
$$W = (\partial\rho_2/\partial\rho_1)$$
$$r = (\partial\rho/\partial s)_p$$

along the shock adiabatic

For a perfect gas, such as we shall consider in this paper, the jump conditions simplify as follows:

$$(\bar{\rho}_2/\bar{\rho}_1) = W = F_2 = (\bar{u}_{1x}/\bar{u}_{2x})$$

$$(\bar{p}_2/\bar{p}_1) = F_3 \qquad (c_2/c_1) = R = F_4$$

$$c_2^2 M_{2n}^2 Q = 1/M_{1n}^2 \qquad c_2^2 M_{2n}^2 P = F_5 \quad (13b)$$

As shown in Fig. 1, our reference coordinate system is taken parallel and normal to the unperturbed plane shock wave. However, during the interaction between a shock wave and some small-amplitude incident disturbance, the shock wave may be deformed. We assume that the disturbance wave amplitude is very small compared to the shock strength so that the shock deformation is small and may be characterized by a ripple propagating along the shock line and an oscillation of the shock line along the x axis (the normal to the unperturbed shock line). Further, we shall assume that the shock deformation is of the same form as the incident disturbance wave, that is, plane harmonic.

With the above assumptions we can readily determine the normal vector to the deformed shock, the oscillation velocity u_s' of the distorted shock and the perturbed and unperturbed velocities of the medium relative to the deformed shock, each in terms of the shock deformation. Employing these in the continuity and momentum relations of Eqs. (12) we obtain the following small-perturbation jump relations for the interaction of a shock wave with a small-disturbance plane wave

$$(\bar{u}_{2x}/c_2^2)p_2' + \bar{\rho}_2 u_{2x}' + [\bar{\rho}](1 - u_y k_y/\omega)u_s' + \bar{u}_{2x} r_2 s_2'$$

$$= (\bar{u}_{1x}/c_1^2)p_1' + \bar{\rho}_1 u_{1x}' + \bar{u}_{1x} r_1 s_1'$$

$$(1 + M_{2n}^2)p_2' + 2qu_{2x}' + \bar{u}_{2x}^2 r_2 s_2'$$

$$= (1 + M_{1n}^2)p_1' + 2qu_{1x}' + \bar{u}_{1x}^2 r_1 s_1'$$

$$u_{2y}' + [u_x](k_y/\omega)u_s' = u_{1y}' \tag{14}$$

where q = normal mass flux $(\bar{\rho}\bar{u}_n)$, (ω/k_y) = the phase speed of a ripple propagating along the shock, u_s' = the x component of the velocity of the deformed shock, and M_{1n}, M_{2n} are the normal components of the unperturbed Mach numbers on the fore and aft sides of the shock, respectively. These relations in conjunction with Eqs. (13) suffice to compute the jump and/or generation of plane acoustic, entropy, and vorticity waves across a shock, as well as the shock oscillation u_s' due to interaction between the shock wave and the plane disturbances. Using Eqs. (13) to eliminate the entropy fluctuations behind the shock in Eqs. (14), the small-perturbation amplitude jump relations may be neatly expressed in the following simple matrix form

$$\begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} (p_2', U_2', u_s') = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \tag{15}$$

where p_2', $U_2'(\equiv\sqrt{u_{2x}'^2 + u_{2y}'^2})$, and u_s' are, respectively, the amplitudes of the plane acoustic wave, the plane vorticity wave behind the shock, and the shock speed.

$$g_{11} = (\alpha_2 + c_2^2 M_{2n} Q)/c_2;\quad g_{12} = -\bar{\rho}_2\beta_3;\quad g_{13} = [\bar{\rho}](1 - u_y k_y/\omega)$$

$$g_{21} = 1 + 2\alpha_2 M_{2n} + 1/M_{1n}^2;\quad g_{22} = -2q\beta_3;\quad g_{23} = 0$$

$$g_{31} = \beta_2/(\bar{\rho}_2 c_2);\qquad g_{32} = \alpha_3;\qquad g_{33} = [\bar{u}_x]k_y/\omega$$

The input functions X_1, X_2, and X_3 depend on the particular incident plane wave as follows:

1) For plane acoustic wave incident from ahead of shock

$$X_1 = p_1'[\alpha_1/c_1 + M_{1n}/c_1 - c_2 M_{2n} P]$$

$$X_2 = p_1'[1 + 2\alpha_1 M_{1n} + M_{1n}^2 - c_2^2 M_{2n}^2 P] \tag{16}$$

$$X_3 = p_1'[\beta_1/(\bar{\rho}_1 c_1)]$$

2) For plane acoustic wave incident from behind the shock

$$X_1 = -\bar{p}'[\alpha_1/c_2 + c_2 M_{2n} Q]$$

$$X_2 = -\bar{p}'[1 + 2\alpha_1 M_{2n} + 1/M_{1n}^2]$$

$$X_3 = -\bar{p}'[\tilde{\beta}_1/(\bar{\rho}_2 c_2)] \tag{17}$$

3) For plane entropy wave incident from ahead of shock

$$X_1 = X_3 = 0$$

$$X_2 = -(s_1'/c_p)\bar{\rho}_1 c_1^2 M_{1n}^2 (1 - \bar{\rho}_1/\bar{\rho}_2) \tag{18}$$

4) For plane vorticity wave incident from ahead of shock

$$X_1 = -U_1'\beta_1\bar{\rho}_1$$

$$X_2 = -2U_1'\bar{\rho}_1\beta_1 c_1 M_{1n} \tag{19}$$

$$X_3 = U_1'\alpha_1$$

We are interested in the vorticity wave transmission and generation; for this, Eq. (15) yields

$$U_2' = \{-X_1(g_{21}g_{33} - g_{31}g_{23}) + X_2(g_{11}g_{33} - g_{31}g_{13})$$

$$- X_3(g_{11}g_{23} - g_{21}g_{13})\}/\Delta \tag{20}$$

where $\Delta = [\bar{\rho}]F_7$. Discussion of the implication of $\Delta = 0$ is given in Ref. 21.

Plane Vorticity Wave Generation and Transfer Functions

We define a generation (or transfer) function as the ratio of the amplitude of the generated (or transmitted) plane disturbance behind a shock wave to the amplitude of the incident plane disturbance, where the numerator and denominator are separately nondimensionalized with appropriate unperturbed medium flow variables ahead of the shock. Thus, for instance, the generation function for plane vorticity waves produced by an incident plane acoustic wave would be $G_P = U_2'/(\bar{u}_1 p_1'/\bar{\rho}_1)$.

From the plane disturbance jump relation in foregoing sections we then obtain the following generation and transfer functions for the plane vorticity wave.

1) Incident acoustic wave from ahead of shock

$$G_P \equiv U_2'/(\bar{u}_1 p_1'/\bar{\rho}_1) = [H_1\{(\alpha_1 + M_{1n})/F_2 - F_5/M_{1n}\}$$

$$- H_2 F_1(1 + 2\alpha_1 M_{1n} + M_{1n}^2 - F_5) + H_3\beta_1]/(\gamma M_1 F_7) \tag{21}$$

from behind the shock

$$G_{\bar{p}} \equiv U_2'/(\bar{u}_1\bar{p}'/\bar{\rho}_1) = [H_1(1 + \alpha_1 F_1 M_{1n}^3) + H_2 F_1 M_{1n}$$

$$\times (1 + 2\alpha_1 F_1 M_{1n}^3 + M_{1n}^2) - H_3 F_1 M_{1n}^3 \beta_1]/(\gamma M_1 M_{1n}^3 F_7) \tag{22}$$

2) Incident entropy wave

$$G_E \equiv U_2'/(\bar{u}_1 s_1'/c_p) = H_2 F_1 M_{1n}^2 (F_2 - 1)/(M_1 F_2 F_7) \tag{23}$$

3) Incident vorticity wave

$$G_V \equiv U_2'/U_1' = \{\beta_1(2F_1 M_{1n} H_2 - H_1/F_2) + \alpha_1 H_3\}/F_7 \tag{24}$$

Our x-y coordinates are not necessarily the coordinate directions x'-y' parallel and normal to the direction of the flow of the unperturbed medium. We have presumed in this paper that the flow of the unperturbed medium is always parallel to x' and that the medium has no velocity component in the y' direction. Therefore, for an oblique shock we have utilized a new coordinate system x-y which is a rotation of x'-y' through the angle $(90\text{-}\theta_s)$ where θ_s is the shock angle.

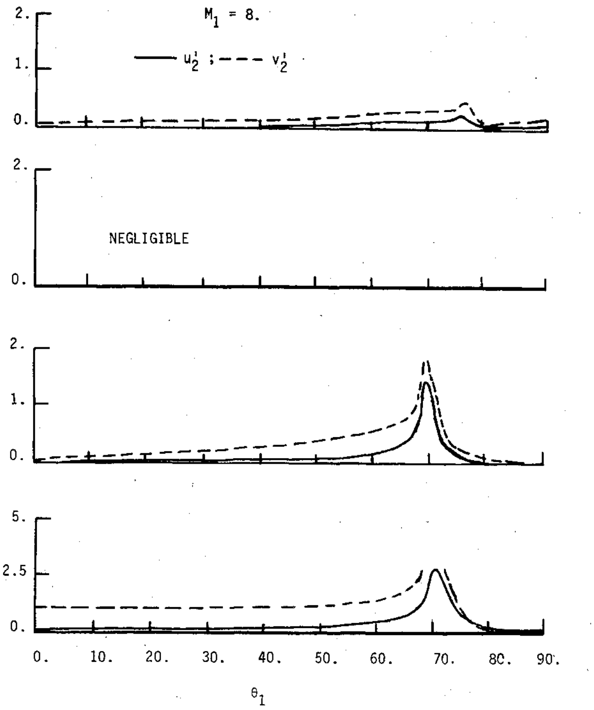

Fig. 2 Plane vorticity wave transfer/generation function dependence on incident disturbance wave angle.

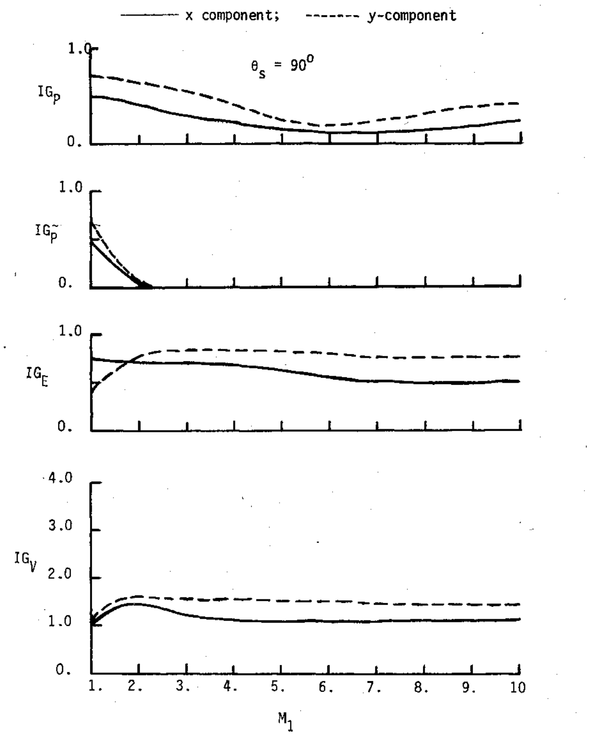

Fig. 3 Turbulence transfer/generation function dependence on incident freestream Mach number, normal shock case.

Such a rotation makes the shock normal, with both a parallel and a crossflow of the medium across the shock. The perturbed velocity vector U_2' associated with a transmitted or generated plane vorticity wave behind the shock may therefore be split into components u_2' parallel to and v_2' perpendicular to the unperturbed flow direction of the medium, where

$$|u_2'| = U_2' \cos(\theta_s + \theta_3)$$
$$|v_2'| = U_2' \sin(\theta_s + \theta_3) \quad (25)$$

Figure 2 shows plots of the plane vorticity wave generation and transfer functions for the above two components. The scaling factors remain as in Eqs. (21-24); thus, we would have the transfer functions $|u_2'|/U_1'$ and $|v_2'|/U_1'$.

Peaks occur in the generation and transfer functions for plane vorticity waves at the critical angles of incidence. In some cases the generation function may have more than one peak. Cuadra[20] observed and discussed similar peakiness in the generation and transfer functions.

Turbulence Generation and Transfer Functions

Given the transfer and generation functions for the plane vorticity wave across a shock and in conjunction with our assumption that incident and refracted plane disturbance waves must have the same frequency and the same y component of the disturbance wave vector, we may obtain the turbulence transfer and generation functions by integration over all incident angles in the plane and over all frequencies. Ribner[12,18] has discussed this integration process in some detail.

Recall that G_V [Eq. (24)] is the transfer function (U_2'/U_1') relating velocities downstream and upstream of the shock for an incident plane oblique vorticity wave. Following Ref. 18, the corresponding relation between downstream and upstream turbulence would be

$$U_2'/U_1' = (3/2)\int_0^{\pi/2} |G_V|^2 \sin^3\theta_1 \, d\theta_1 \quad (26)$$

This is based on the assumption that the three-dimensional wave-number spectrum of the upstream turbulence has a form appropriate to isotropic turbulence. Similar relations may be written when the incident disturbances are sound waves or entropy waves; such relations would involve the assumed spectrum of the disturbances as well as the appropriate transfer functions. In the calculations reported herein we have assumed that the input spectrum is such that an equation of the form of Eq. (26) still applies, with $|G_V|$ replaced by $|G_P|$, $|G_{\bar{P}}|$, or $|G_E|$, as appropriate.

We denote these turbulence (integrated spectra) generation or transfer functions by IG_p, IG_P, $IG_{\bar{P}}$, IG_E, and IG_V for sound waves incident from ahead, sound waves incident from behind, incident entropy waves, and incident vorticity waves, respectively. Figures 3 and 4 show plots of these turbulence generation and transfer functions for several incident stream Mach numbers in air and for a normal ($\theta_s = 90$ deg) and an oblique ($\theta_s = 30$ deg) shock.

Turbulence Amplifier-Generation Mechanisms

Based upon the linearized analyses of the foregoing sections as well as upon some experimental data,[10,22-24] we suggest three major mechanisms which appear to be responsible for the significant generation of turbulence behind a shocked region of a shear flow. These are: 1) direct amplification of incident turbulence across a shocked region, 2) generation of turbulence from incident acoustic and entropy fluctuation modes, and 3) "pumping" of turbulence from the mean flow by "externally driven" shock oscillation. There are several secondary turbulence amplifier mechanisms such as, for instance, the focusing of higher frequency turbulence by large concave distortions of the shock caused by lower frequency fluctuations[25]; we do not discuss these secondary mechanisms in this paper.

Let us now very briefly consider each of the primary turbulence amplifier-generator mechanisms.

Direct Turbulence Amplification (Mechanism 1)

Figures 3 and 4 indicate that turbulence incident upon a shock wave can be amplified by up to 30% across the shock. In a shocked region of a shear flow, there will usually be several orders of shock compression and expansion wave surfaces, across which several orders of refraction may easily lead to larger amplification than the 30% suggested by linear theory for a single shock wave.

The turbulence amplification is seen to be critically dependent on shock wave angle.

Turbulence Generation from Acoustic and Entropy Mode Transfer (Mechanism 2)

Figures 3 and 4 indicate that acoustic and entropy fluctuation modes incident on a shock wave can generate vorticity fluctuation levels up to 50% of the intensity of the incident mode. The entropy fluctuation mode appears to generate more turbulence than the acoustic mode under given shock-wave conditions.

Again, across a shocked region of a shear layer several orders of shock and compression wave surfaces are usually present, across and on which several orders of refraction, reflection, generation, and rereflection of fluctuation modes may easily create turbulent intensities far in excess of values incident upon the shocked region.

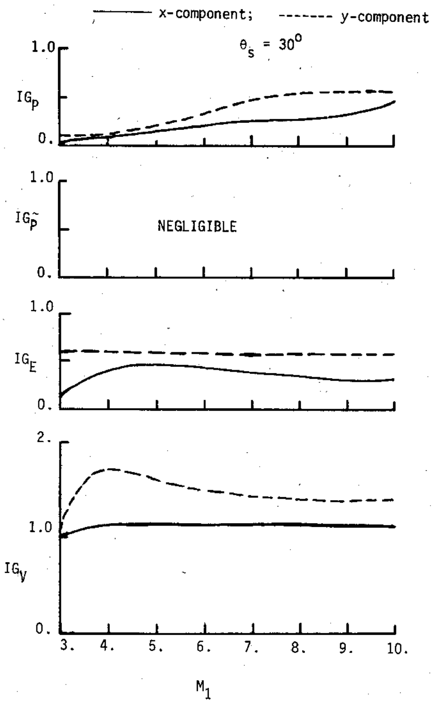

Fig. 4 Turbulence transfer/generation function dependence on incident freestream Mach number, oblique shock case.

Turbulence "Pumping" by Externally Driven Shock Oscillation (Mechanism 3)

Equations (13) and (14) present the relations between the magnitudes of plane acoustic, entropy, and vorticity waves ahead of and behind a shock wave, as well as the oscillation induced upon the shock wave by its interaction with the plane disturbance wave. If we imagine that the shock oscillation is externally induced and therefore consider the shock oscillation terms in Eqs. (14) to be known quantities, then by use of Eqs. (13) we may eliminate the acoustic and entropy terms behind the shock from Eqs. (14). We are left with an equation of the form

$$(U_2'/\bar{u}_1) = B_0(u_s'/\bar{u}_1) + B_1(p_1'/\bar{p}_1) + B_2(s_1'/c_p) + B_3(U_1'/\bar{u}_1) \tag{27}$$

which predicts the total plane vorticity wave generated behind a shock as a result of shock oscillation, incident plane acoustic wave, incident plane entropy wave, and incident plane vorticity wave.

Equation (27) implies that an externally driven shock oscillation would, even in the absence of any incident disturbance waves, generate turbulence behind the shock wave. The generation function for such a situation may be written for the case of a "rigid" shock oscillating with speed u_s' in the x coordinate direction as

$$(U_2'/u_s') = 2(M_{1n}^2+1)/\{(\gamma+1)M_{1n}^2\} \tag{28}$$

Morkovin[14] formulated a similar generation function for a normal shock wave oscillating parallel to the mean flow direction and with no rotation. He obtained

$$(U_2'/u_s') = (1-\bar{\rho}_1/\bar{\rho}_2)\{1+M_{2n}^2+(\gamma-1)$$
$$\times(1-\bar{\rho}_2/\bar{\rho}_1)M_{2n}^2\}/(1-M_{2n}^2) \tag{29}$$

Equations (28) and (29) are identical. Typical results are shown in Fig. 5 which plots (U_2'/u_s') as a function of incident Mach number M_1 for the normal shock case. Except at transonic speeds (M_1 in the neighborhood of 1), $(U_2'/u_s') \simeq 83\%$, independent of the incident Mach number. At transonic speeds externally driven shock oscillation can generate turbulence of intensity in excess of 150% of the shock oscillation intensity.

Typical mechanisms giving rise to "externally driven" shock oscillations include: 1) reflection of the incident shock from a "sonic line" whose location is variable in space-time due to the presence of turbulence, and 2) low-frequency "breathing" of the separated flow region.

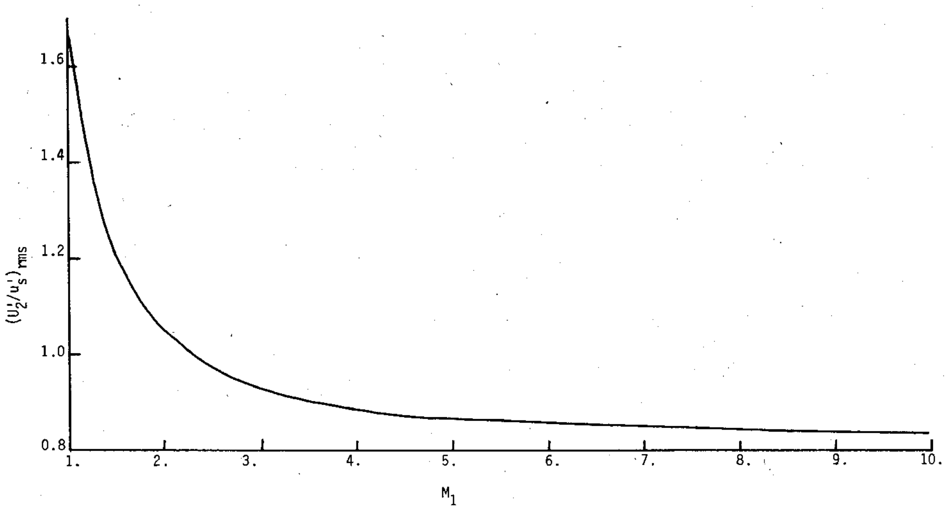

Fig. 5 Turbulence generation function due to externally driven shock oscillation.

Conclusion

Simple, linearized analyses in conjunction with available experimental data have shown that, contrary to current low-speed rapid distortion theories, turbulence can amplify several-fold across a shocked region. The theory has identified several mechanisms which appear capable of causing in excess of a 100% turbulence amplification across shocked regions. Furthermore, the theory has identified critical parameters which influence such turbulence amplification: 1) the incident Mach number M_1, 2) the shock wave angle θ_s, 3) the shock oscillation speed u_s', and, of course 4) the medium itself as reflected in the specific heats ratio γ. Some of these critical influencing parameters may themselves depend on the shear layer flow parameters in the shocked region, on whether or not flow separation has occurred in the shocked region, as well as on details of the quasi-inviscid shock-wave field.[26]

The actual physical problem in nonlinear and, especially for transonic flow facilities, is further complicated by the presence of incoming pressure disturbances from the freestream[27] and first-order local shock geometry perturbations due to the incoming disturbance field (results in Figs. 3 and 4 were obtained with the assumption of strong shocks and weak disturbances). Therefore, the present computational approach can provide only crude rules of thumb as to expected amplifications. Of further concern is the probable distortion of the turbulence spectra by these amplification processes. Implicit in most current turbulence closure schemes is the assumption of spectral similarity. Violations of this assumption generally manifest themselves as "variable constants" in the closure equations.

Finally, the present paper indicates that the application of linearized analysis of general fluctuational motion, such as has been developed in acoustic research, to the shock/boundary-layer interaction problem provides reasonable explanations for the experimentally observed large turbulence amplification occurring across shock waves incident upon turbulent boundary layers. As a zero-order approach these amplification factors could be employed locally in large-scale numerical calculations to "correct" the usual low-speed closure approaches for shock amplification processes.

Authors' Note

Equations (3a), (3b), (17b), (24), and (29) of Ref. 19 contain typographical errors which have been corrected in the present work. Turbulence transfer functions computed using the corrected form of Ref. 19 agree very well with similar results presented by Ribner[13] and by Kerrebrock.[16] Spot checks were also made among similar results on plane disturbance wave transfer functions obtained by Refs. 13, 14, 16, 19, and 20. We found that once the same scaling is used to represent the transfer functions, the results were almost identical. Hence our confidence in employing Ref. 19 as a representative analysis of the subject problem.

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